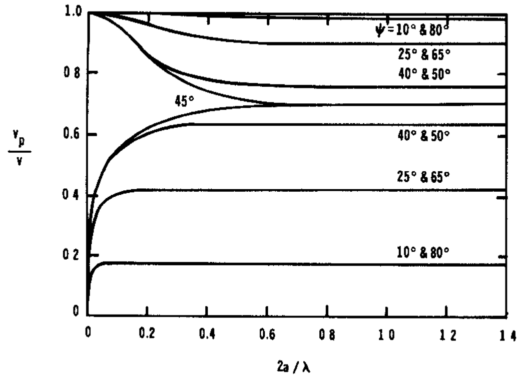

Fig. 3. The same as Fig. 2 except for Case 3). The ratio B_1/A_1 is now determined by the guide parameters.

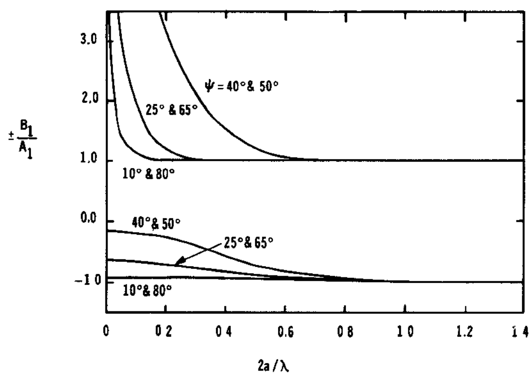

Fig. 4. The ratio B_1/A_1 is plotted as a function of $2a/\lambda$ corresponding to Fig. 3. Plus sign for $\Psi < 45°$, minus for $\Psi > 45°$.

In general, external current return paths at $y = \pm\infty$ are necessary as for one of the two modes for Cases 1) and 2).

The Poynting vectors P_{z1} and P_{y1} are in general ($\Psi \neq 45°$) a linear combination of odd and even functions in x. The wave front is always perpendicular to the z-direction. Neither the spatial integral over area xy of P_{z1} nor that over area xz of P_{y1} is zero in general. The latter is different from Case 2). In the limit that $2a \gtrsim \lambda$, the power flow ratio connected with the lower branch in Fig. 3 ($0 < \Psi < 45°$) is

$$P_{z1}/P_{y1} = \tan\Psi$$

while that connected with the upper branch ($0 < \Psi < 45°$) is

$$P_{z1}/P_{y1} = \tan(\Psi + 90°).$$

In that limit, the power flow connected with the lower branch flows along the slots of the top plate and its power density falls off exponentially from it toward the other plate while the power flow connected with the mode of the upper branch flows along the slots in the bottom plate and its density falls off exponentially toward the top plate. When $2a < \lambda$, these two surface-like modes become more strongly affected by the boundary conditions on the opposite plate and the Poynting vector for each mode is rotated away from the directions of the slots.

Discussion and Conclusion

All three cases have two slow-wave modes. For Case 1), the phase velocity of the two modes is the same and independent of $2a/\lambda$. In principle, they exist for all frequencies within the context of the anisotropic sheath model. For Cases 2) and 3), each mode has, in general, a different phase velocity at a given frequency. For both cases, $v_p/v \to 1$ for one mode and $v_p/v \to 0$ for the other mode when the signal frequency approaches zero. For wavelengths smaller than the separation of the two anisotropic sheaths of the guide, the phase velocities become frequency independent. In Case 2) they approach each other, while in Case 3) they stay separated, except for $\Psi = 45°$, in which instant Case 3) reduces to Case 2). Thus, for Case 3) there are regions of phase velocities for which neither of the two modes can propagate at any frequency ($\Psi \neq 45°$), and the separate modes remain distinct even at high frequencies. Within the present model all phase velocities in the above structures are smaller than v except for $\lambda \to \infty$ where for one of the modes of Cases 2) and 3), $v_p \to v$.

It is important to realize that when the y-axis is not bisecting the angle between the top and bottom slots ($\delta \neq \Psi$) that the wavefront is perpendicular to the z-direction and propagating along it while the Poynting vector for each mode is along the slots for $2a \gtrsim \lambda$ and the wave is of a surface-like nature. The mode of the upper branch in Fig. 3 clings to the bottom plate while the lower branch clings to the top plate ($0 < \Psi < 45°$). Each mode is a linear combination of spatially odd and even terms whose amplitude ratio is fixed by the angles δ and Ψ.

One of the two mode types of Cases 1) and 2) and the single modes in Case 3) ($\Psi \neq 45°$) require at $y = \pm\infty$ current returns which are something other than just direct connection at the edges.

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A Method to Generate Conservation Laws for Coupled Transmission Systems

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***Abstract*—A systematic method is presented for generating a set of conservation laws for spatially distributed coupled linear systems. In contrast with previous practice, where energy balance equations were obtained by manipulating the fundamental equations of the interaction (the Maxwell equations or the equations of mechanics), or by determining the invariant**

Manuscript received October 12, 1981; revised June 22, 1982. This work was supported in part by the Natural Sciences and Engineering Research Council, Canada.

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quadratic forms of the motion, here the coupled systems equations are used as the point of departure. The results apply to lossy as well as to lossless devices. Illustrative examples examine the acoustic power flow in the surface acoustic-wave grating reflector and TE–TM-mode coupling in anisotropic dielectric waveguides.

I. Introduction

The usefulness of conservation laws reaches beyond the physical picture provided by them about the energy balance of a closed system. These laws are also used in conjunction with perturbational and variational techniques [1] to obtain stationary formulas for the propagation constant of a transmission system, or for the resonance frequency of a cavity resonator. Methods to generate conservation laws from the fundamental equations characterizing the interaction have been described in the literature [2], [3]. The ubiquitous Poynting theorem, for example, is obtained by manipulating the Maxwell equations, an analogous conservation law for elastic waves is based on the equations of motion in solids [4, ch. 5].

Pease [5] investigated codirectional and contradirectional coupled systems and found fundamental conditions which must be satisfied by the coupling and transfer matrices characterizing a conservative system. These conditions involve so-called system metrics; appropriate matrices which ensure the temporal or spatial invariance of power flow, expressed in a quadratic form. The metrics, however, are not known *a priori*; the search for them can prove to be difficult.

In this paper, a simple method is presented to obtain a set of conservation laws without having to determine the system metrics. The price to pay for this convenience is that the resultant expressions contain not only the components of the state vector but their derivatives as well. Thus, the conservation law applies to a localized point in space or time, rather than to the global interaction and, consequently, involves the coupling matrix which describes the evolution of the state vector. There are no restrictions placed on the coupling matrix, therefore the method is equally applicable to lossless or lossy, to uniform or nonuniform, to codirectional or contradirectional couplers.

Section II reviews the conservation laws expressed in terms of metrics, Section III introduces the proposed new method, and finally, Section IV illustrates the new method by way of examples drawn from ultrasonics and electromagnetics.

II. Conservation Laws

Consider the uniform, linear, distributed system characterized by

$$\partial \bar{a}(x)/\partial x = -jR\bar{a}(x) \tag{1}$$

where x is the spatial coordinate, $\bar{a}^T = [a_1, a_2]$ is the two-component state vector, and R is the system coupling matrix. Assume, for the present, that R describes a lossless codirectional coupler. Factoring $-j$ in (1) simplifies the system matrix of lossless couplers whose spatial dependence includes the $\exp(-j\bar{k}\cdot\bar{r})$ factor.[1]

It is well known [6] that conservation of power in this case is expressed by

$$\partial\left(|a_1|^2 + |a_2|^2\right)/\partial x = 0 \tag{2}$$

signifying the invariance of the sum of the modal powers along the direction of propagation. It is much less recognized that the same system complies with a set of conservation laws

$$q_i = \bar{a}^\dagger K_i \bar{a} = \text{const.} \tag{3}$$

enumerated by the subscript i, where the † denotes Hermitian conjugation and K_i is a 2×2 matrix, also known as the metric of the system. Equation (3) is a quadratic form which is independent of x only when the kernel is an appropriate metric.

The conditions a valid metric must satisfy can easily be obtained [5], either from (1) or from the system transfer matrix $M(x)$ defined by

$$\bar{a}(x) = M(x)\bar{a}(0). \tag{4}$$

On the one hand, differentiation of (3) with respect to x and subsequent substitution of (1) results in the first condition

$$R^\dagger K_i = K_i R \tag{5}$$

while, on the other hand, direct substitution of (4) into (3) provides the second condition

$$K_i = M^\dagger K_i M. \tag{6}$$

Expressions (5) and (6) are equivalent in the sense that the same metric satisfies both. For any given system there is an infinite number of valid metrics, although only a few are considered useful. For example, the four basic metrics of a lossless forward coupler having a real coupling coefficient are [7]

$$K_0 = I \qquad K_1 = \frac{1}{X}\begin{bmatrix} Y & 1 \\ 1 & -Y \end{bmatrix} \qquad K_2 = \frac{1}{2X}\begin{bmatrix} X+Y & 1 \\ 1 & X-Y \end{bmatrix} \qquad K_3 = I - K_2 \tag{7}$$

where I is the identity, $Y = 1/2|(R_{11} - R_{22})/R_{12}|$ is the asynchronism parameter, and $X = (Y^2+1)^{1/2}$.

It can be shown [8, ch. 9] that the conditions analogous to (5) and (6), applicable to lossless backward couplers are $R^{\#}K_i = K_i R$ and $K_i = M^{\#}K_i M$, respectively, where the adjoint operation denoted by the # superscript is defined by

$$A^{\#} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} A^\dagger \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}. \tag{8}$$

The search for a metric which satisfies (5) or (6), or their backward wave analogues, can be quite laborious, especially when R and M are larger than 2×2. A simpler method to generate a set of conservation laws is described in the next section.

III. Analysis

Consider first the two-dimensional linear system (1) and recall the Pauli spin matrices σ_i, $i = 0,1,2,3$ defined by [9]

$$\sigma_0 = I \qquad \sigma_1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \qquad \sigma_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad \sigma_3 = \begin{bmatrix} 0 & -j \\ j & 0 \end{bmatrix} \tag{9}$$

where σ_1, σ_2, and σ_3 are known to have the following properties: $\sigma_i^\dagger = \sigma_i$, $\sigma_i^2 = I$, $Tr(\sigma_i\sigma_j) = 2\delta_{ij}$, $\sigma_i\sigma_j = -\sigma_j\sigma_i$, and $\sigma_i\sigma_j = j\sigma_k$, i, j, k in cyclic order. To construct a conservation law, determine

$$\bar{a}^\dagger\sigma_i\bar{a}' = -j\bar{a}^\dagger\sigma_i R\bar{a} \tag{10}$$

and its complex conjugate

$$\left(\bar{a}^\dagger\sigma_i\bar{a}'\right)^\dagger = j\bar{a}^\dagger R^\dagger\sigma_i\bar{a} \tag{11}$$

then add and subtract them. The desired expressions are

$$2\,\text{Re}\left(\bar{a}^\dagger\sigma_i\bar{a}'\right) + j\bar{a}^\dagger\left(\sigma_i R - R^\dagger\sigma_i\right)\bar{a} = 0, \qquad i = 0,1,2,3 \tag{12}$$

[1] The corresponding temporal differential equation for systems exhibiting $\exp(j\omega t)$ time dependence is $\partial\bar{a}(t)/\partial t = jW\bar{a}(t)$

and

$$2\,\mathrm{Im}\left(\bar{a}^{\dagger}\sigma_i\bar{a}'\right)+\bar{a}^{\dagger}\left(\sigma_i R + R^{\dagger}\sigma_i\right)\bar{a}=0, \qquad i=0,1,2,3 \quad (13)$$

representing a total of eight real conservation laws. Note that in optics, $\bar{a}^{\dagger}\sigma_i\bar{a}$ is known as the ith component of the Stokes vector $\bar{s}(z)$ [9]. Therefore, instead of (12), one can also write

$$s_i' + j\bar{a}^{\dagger}\left(\sigma_i R - R^{\dagger}\sigma_i\right)\bar{a}=0. \quad (14)$$

Conservation laws for larger coupled systems, whose dimensionality is divisible by two, can also be generated by the above method using the Kronecker or direct product [8, ch. 14]. For example, in a four dimensional system, the expression

$$\bar{a}^{\dagger}\left(\sigma_i\times\sigma_j\right)\bar{a}' = -j\bar{a}^{\dagger}\left(\sigma_i\times\sigma_j\right)R\bar{a} \quad (15)$$

and its complex conjugate can be used in place of (10) and (11). Subsequent addition and subtraction provide the desired conservation laws which are formally identical to (12) and (13) if σ_i is replaced by $\sigma_i\times\sigma_j$ everywhere.

A note of caution regarding the evaluation of (12) and (13). When R includes a differential operator one must remember that in the $\bar{a}^{\dagger}R^{\dagger}\sigma_i\bar{a}$ term this differential operator operates on the elements of $\bar{a}^{\dagger}$, rather than on those of $\bar{a}$. For example, when

$$\bar{a}=\begin{bmatrix} a_1 \\ a_2\end{bmatrix} \qquad \sigma_i=\sigma_0 \text{ and } R=\begin{bmatrix}\partial/\partial y & 0\\ 0 & -\partial/\partial y\end{bmatrix}$$

then

$$\bar{a}^{\dagger}R^{\dagger}\sigma_0\bar{a}=a_1^*\partial a_2^*/\partial y - a_2^*\partial a_2^*/\partial y \quad (16)$$

whereas

$$\bar{a}^{\dagger}\sigma_0 R\bar{a}=a_1^*\partial a_1/\partial y - a_2^*\partial a_2/\partial y \quad (17)$$

the asterisk indicating complex conjugation.

In the foregoing, no restriction has been placed on R. Accordingly, the described method is applicable to lossy systems as well as to lossless ones, to-backward couplers as well as to forward couplers. In fact, R may itself depend on x as in tapered, chirped, or periodically coupled devices.

IV. Applications

A few examples will now be considered.

Example 1: Surface Acoustic Wave Grating Reflector

The system matrix characterizing a "wide" surface acoustic wave grating reflector is [10]

$$R=\begin{bmatrix}\delta+(1/2k_0)\,\partial^2/\partial y^2 & \kappa\\ -\kappa^* & -\delta-(1/2k_0)\,\partial^2/\partial y^2\end{bmatrix} \quad (18)$$

where $\delta=[(\omega-j\sigma)-\omega_0]/v$ is the complex detuning parameter, κ is the complex coupling coefficient between the forward and the backward traveling waves, ω_0 is the Bragg frequency, v is the phase velocity of the x directed Rayleigh wave in the nondispersive approximation, $k_0=\omega_0/v$, and diffraction across the width of the device is accounted for by the $(1/2k_0)\,\partial^2/\partial y^2$ term.

The conservation laws corresponding to $i=0$ and 1 in (12) are in this case

$$\frac{\partial}{\partial x}\left[|a|^2-|b|^2\right]+\frac{\partial}{\partial y}\left[\frac{\beta_y}{k_0}\left(|a|^2+|b|^2\right)\right]=2\left[|a|^2+|b|^2\right]\mathrm{Im}\,\delta \quad (19)$$

and

$$\frac{\partial}{\partial x}\left[|a|^2-|b|^2\right]+\frac{\partial}{\partial y}\left[\frac{\beta_y}{k_0}\left(|a|^2-|b|^2\right)\right]$$
$$=2\left[|a|^2-|b|^2\right]\mathrm{Im}\,\delta+4\,\mathrm{Re}(ab^*)\,\mathrm{Im}\,\kappa-4\,\mathrm{Im}(ab^*)\,\mathrm{Re}\,\kappa \quad (20)$$

where a and b are the normalized forward and backward traveling wave amplitudes, respectively, and $\partial/\partial y=-jk_y=-\alpha_y-j\beta_y$.

Recognizing that $|a|^2-|b|^2$ is the net surface power flow density (power per unit width) in the x direction, that $\mathrm{Im}\,\delta=\sigma/v$, where σ is the exponential decay rate, and that $(|a|^2+|b|^2)/v$ is the surface energy density (energy per unit area) in the device, (19) is immediately identified as Poynting's theorem. Equation (20) is complementary to (19) in the sense that while the *l.h.s.* of (19) describes the two-dimensional divergence of $\bar{p}=[|a|^2-|b|^2]\hat{a}_x+(\beta_y/k_0)[|a|^2+|b|^2]\hat{a}_y$, (20) is the corresponding divergence of $\bar{q}=[|a|^2+|b|^2]\hat{a}_x+(\beta_y/k_0)[|a|^2-|b|^2]\hat{a}_y$. Either of these expressions can be used in a variational principle to determine, e.g., the resonance frequency of an eigenmode in a Fabry–Perot resonator [10].

Carrying out the differentiation with respect to y, the four conservation laws contained in (12) result in

$$\frac{d\bar{s}}{dx}=2\begin{bmatrix}0 & \mathrm{Im}\,\delta+\alpha_y\beta_y/k_0 & \mathrm{Im}\,\kappa & \mathrm{Re}\,\kappa\\ \mathrm{Im}\,\delta+\alpha_y\beta_y/k_0 & 0 & 0 & 0\\ \mathrm{Im}\,\kappa & 0 & 0 & -\mathrm{Re}\,\delta+\mathrm{Re}k_y^2/2k_0\\ \mathrm{Re}\,\kappa & 0 & \mathrm{Re}\,\delta-\mathrm{Re}k_y^2/2k_0 & 0\end{bmatrix}\bar{s} \quad (21)$$

where $s_i=[a^*,b^*]\sigma_i\left[\begin{smallmatrix}a\\b\end{smallmatrix}\right]$. In a coupler where a and b represent orthogonally polarized waves (21) determines the evolution of polarization along the axial coordinate [11].

Example 2: Lossless Anisotropic Dielectric Slab Waveguide

It has been shown [12] that in each homogeneous region of a lossless, anisotropic dielectric slab waveguide Maxwell's equations reduce to (1) where x is the coordinate normal to the interfaces, perpendicular to the direction of propagation z, $\bar{a}^T=[E_y,\eta_0H_z,E_z,-\eta_0H_y]$ is the state vector of the field components lying in the plane of the interface, $\eta_0=(\mu_0/\epsilon_0)^{1/2}$, and R is a 4×4 constant matrix whose elements contain the axial wavenumber k_z and the relative permitivities of the region ϵ_{ij}, $i,j=x,y,z$. The diagonal blocks of R describe, respectively, TE- and TM-wave propagation in the guide. Off-block diagonal elements represent TE–TM coupling. In case of uniaxial dielectrics in what is known as the longitudinal or polar configuration, a reference to the optic axis lying in the $x-y$ or $y-z$ plane, respectively, (1) can be transformed into a very simple set of second-order differential equations. In the polar case, for example, one obtains

$$\partial^2\bar{E}_\tau/\partial x^2=-A\bar{E}_\tau \text{ and } \partial^2\bar{H}_\tau/\partial x^2=-A^T\bar{H}_\tau \quad (22)$$

where

$$\bar{E}_\tau=\begin{bmatrix}E_y\\E_z\end{bmatrix} \qquad \bar{H}_\tau=\begin{bmatrix}H_z\\-H_y\end{bmatrix}$$

and

$$A=\begin{bmatrix}k_0^2\epsilon_{yy}-k_z^2 & k_0^2\epsilon_{yz}\\ k_0^2\epsilon_{yz}-k_z^2\epsilon_{yz}/\epsilon_{xx} & k_0^2\epsilon_{zz}-k_z^2\epsilon_{zz}/\epsilon_{xx}\end{bmatrix}.$$

A set of conservation laws can be obtained either by substituting (22) into

$$2\,\mathrm{Re}\left(\bar{E}_\tau^\dagger \sigma_i \bar{E}_\tau''\right) + \bar{E}_\tau^\dagger\left(\sigma_i A + A^T \sigma_i\right)\bar{E}_\tau = 0, \qquad i = 0,1,2,3 \quad (23)$$

and

$$2\,\mathrm{Im}\left(\bar{E}_\tau^\dagger \sigma_i \bar{E}_\tau''\right) - j\bar{E}_\tau^\dagger\left(\sigma_i A - A^T \sigma_i\right)\bar{E}_\tau = 0, \qquad i = 0,1,2,3 \quad (24)$$

where the prime indicates differentiation with respect to x, or by applying the Kronecker product method described in Section III. Since the transverse field vectors in a homogeneous anisotropic dielectric are connected through the wave impedance matrix, $\bar{E}_\tau = \eta_0 Z \bar{H}_\tau$ and, consequently, $\bar{a}$ can be expressed by $\bar{H}_\tau$ alone

$$\bar{a} = \eta_0 P \begin{bmatrix} Z \\ \sigma_0 \end{bmatrix} \bar{H}_\tau \quad (25)$$

where

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

is a permutation matrix, the Kronecker product procedure yields for the first set of conservation laws

$$\frac{\partial}{\partial x}\left[\bar{H}_\tau^\dagger Q \bar{H}_\tau\right] + j\left[\bar{H}_\tau^\dagger V \bar{H}_\tau\right] = 0 \quad (26)$$

where the 2×2 matrices Q and V are

$$Q = \begin{bmatrix} Z \\ \sigma_0 \end{bmatrix}^\dagger P\left(\sigma_i \times \sigma_j\right) P \begin{bmatrix} Z \\ \sigma_0 \end{bmatrix}$$

and

$$V = \begin{bmatrix} Z \\ \sigma_0 \end{bmatrix}^\dagger P\left[\left(\sigma_i \times \sigma_j\right) R - R^\dagger\left(\sigma_i \times \sigma_j\right)\right] P \begin{bmatrix} Z \\ \sigma_0 \end{bmatrix}$$

respectively.

When $-jk_x$ is substituted for $\partial/\partial x$ in (23), four expressions are obtained, each of which depends on k_z^2, $\mathrm{Re}\, k_x^2$, the frequency parameter k_0^2, and the TE–TM field ratio $r = E_z/E_y$ of an elementary wave. These expressions are

$$k_z^2\left[1 + \frac{\epsilon_{yz}}{\epsilon_{xx}}\mathrm{Re}\, r + \frac{\epsilon_{zz}}{\epsilon_{xx}}|r|^2\right] + \mathrm{Re}\, k_x^2\left[1 + |r|^2\right] = k_0^2\left[\epsilon_{yy} + 2\epsilon_{yz}\mathrm{Re}\, r + \epsilon_{zz}|r|^2\right]$$

$$k_z^2\left[1 - \frac{\epsilon_{yz}}{\epsilon_{xx}}\mathrm{Re}\, r - \frac{\epsilon_{zz}}{\epsilon_{xx}}|r|^2\right] + \mathrm{Re}\, k_x^2\left[1 - |r|^2\right] = k_0^2\left[\epsilon_{yy} - \epsilon_{zz}|r|^2\right]$$

$$k_z^2\left[\frac{\epsilon_{yz}}{\epsilon_{xx}} + \left(1 + \frac{\epsilon_{zz}}{\epsilon_{xx}}\right)\mathrm{Re}\, r\right] + \mathrm{Re}\, k_x^2\left[2\,\mathrm{Re}\, r\right] = k_0^2\left[\epsilon_{yz}\left(1 + |r|^2\right) + \left(\epsilon_{yy} + \epsilon_{zz}\right)\mathrm{Re}\, r\right]$$

$$\left[k_z^2\left(1 + \frac{\epsilon_{zz}}{\epsilon_{xx}}\right) + 2\,\mathrm{Re}\, k_x^2 - \left(\epsilon_{yy} + \epsilon_{zz}\right)k_0^2\right]\mathrm{Im}\, r = 0. \quad (27)$$

On the other hand, (26) supplies 16 equations of the type encountered in Example 1, where the spatial rate of change of a power flow density (power per unit area) is compared to the decay rate of the energy density (energy per unit volume). From this large number of conservation laws one must carefully select those which are most useful.

Note that (26) as well as (27) are expressed in terms of field amplitudes. These are therefore potentially suitable to be used in a stationary formula to determine k_z or the impedance parameters for a given frequency [1]. In addition, each of (27) describes an ellipse in terms of the TE–TM field ratio r, whose principal axes are $k_x/k_0 = c/v_{px}$ and $k_z/k_0 = c/v_{pz}$. Since the r is directly related to the angle enclosed between the z direction and the optic axis, this so-called slowness ellipse [4, sec. 7D] provides a useful relationship between crystal orientation and the inclination of angled waves in the guide.

V. Conclusions

A simple method has been introduced to generate conservation laws applicable to linear transmission systems characterized by their coupling matrix. These conservation laws are useful for obtaining stationary formulas for the propagation constant of a waveguide, for the modal resonance frequency of a cavity resonator, for the impedance parameters characterizing a TE–TM mode coupler, or for determining the dependence of angled waves on crystal orientation in an anisotropic slab waveguide, etc. Examples related to acoustic surface wave devices and anisotropic dielectric waveguides illustrate the method.

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Equivalent Circuit of a Gap in the Central Conductor of a Coaxial Line

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Abstract—**The equivalent circuit of a gap in the central conductor of a TEM coaxial line has been determined by the variational technique. Theoretically computed circuit parameters show excellent agreement with the experimental data available in the literature. The gap equivalent circuit**

Manuscript received February 17, 1982; revised April 28, 1982.

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